9 – 3 Graph Rational Functions

rational function $f(x)=\frac{p(x)}{q(x)}$, where p and q are polynomial functions

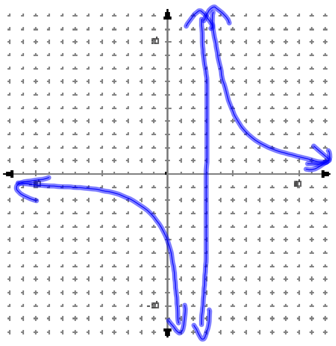
EX: $f(x)=\frac{3}{x-2}$ $g(x)=\frac{x+2}{x^2-4}$ $h(x)=\frac{x^2+4x-12}{x^2+2x-3}$

vertical asymptote (infinite discontinuity) – if the rational expression of a function is in simplest form and the function is undefined for **x = a**, **then x = a is a vertical asymptote.**

EX: $f(x)=\frac{2}{x-3}$

$x \neq 3$

$x=3$ asymptote



hole (point discontinuity) - if **x - a** is a factor of the numerator and denominator of a rational function, then there is a hole in the graph at **x = a.**

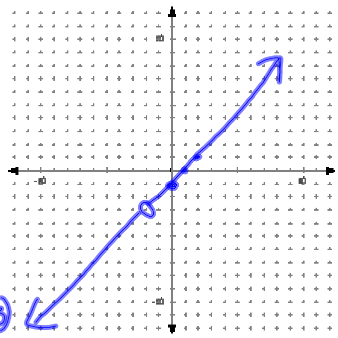
cancel

EX: $f(x)=\frac{x^2+x-2}{x+2}$

$f(x)=\frac{\cancel{(x+2)}(x-1)}{\cancel{(x+2)}}$

$f(x)=x-1$

hole $x=-2$ $(-2,-3)$



horizontal asymptote: Given a rational function $f(x)=\frac{p(x)}{q(x)}$, where p and q are polynomials:

a) If the degree of p is less than the degree of q, then **y = 0 is a horizontal asymptote.**

N < D

b) If the degree of p, with lead coefficient **a,** is equal to the degree of q, with lead coefficient **b,** then $y=\frac{a}{b}$ **is a horizontal asymptote.**

N=D

c) If the degree of p is greater than the degree of q, then there is **no horizontal asymptote.**

N > D

Examples:

1. $f(x)=\frac{2x}{x+3}$

H.A. $y=2$

V.A. $x=-3$

Holes None

2. $f(x)=\frac{x+1}{x^2-1}$ $\quad \frac{\cancel{(x+1)}}{(x-1)\cancel{(x+1)}} \quad \frac{1}{(x-1)}$

(N<D)

H.A $y=0$

V.A. $x=1$

hole $(-1,-\frac{1}{2})$

3. $f(x)=\frac{x^2+5x+6}{x+1}$ $\quad \frac{(x+2)(x+3)}{(x+1)}$

N>D

HA. none

V.A. $x=-1$

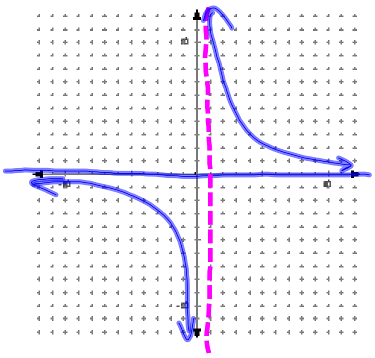
Hole None

1. $f(x)=\frac{3}{x-1}$

HA $y=0$

VA $x=1$

Hole no

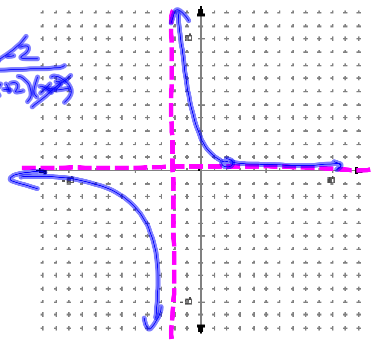


2. $f(x)=\frac{x-2}{x^2-4}$ $\quad \frac{\cancel{x-2}}{(x+2)\cancel{(x-2)}} \quad \frac{1}{x+2}$

HA $y=0$

VA $x=-2$

Hole $(2,\frac{1}{4})$

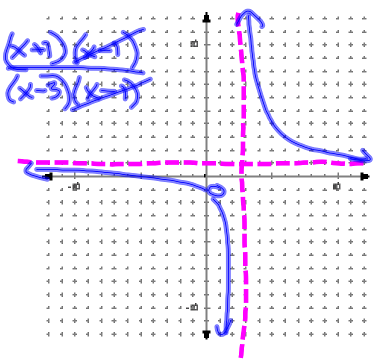


3. $f(x)=\frac{1x^2-1}{1x^2-4x+3}$ $\quad \frac{(x+1)\cancel{(x-1)}}{(x-3)\cancel{(x-1)}}$

HA- $y=1$

VA $x=3$

Hole $(1,-1)$



HW
p489
16-21
23, 25, 31,33 Graphs